Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT3 **Branch: B. Tech (All)**

Semester : 1 **Date : 14/03/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) nth derivative of $y = e^{-x}$ is

(A) $-e^{-x}$ (B) $(-1)^n e^{-x}$ (C) $(-1)^{n+1} e^{-x}$ (D) none of these

b) If $y = \sin 2x \cos 2x$ then y_n equal to

(A) $\frac{1}{2}(4)^n \cos\left(\frac{n\pi}{2} + 4x\right)$ (B) $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 4x\right)$

(C) $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 2x\right)$ (D) none of these

c) If $y = \sin^{-1} x$, then x equal to

(A) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - .....$ (B) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + ...$

(C) $1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + ...$ (D) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - .....$

d) The series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + ...$ represent expansion of

(A) $\sin x$ (B) $\cos x$ (C) $\sinh x$ (D) $\cosh x$

e) $\lim_{x \to 0} \frac{5^x - 3^x}{x} =$ _____

(A) 2 (B) $\log 2$ (C) $\log 15$ (D) $\log\left(\frac{5}{3}\right)$

f) $\lim_{x \to \infty} x^n e^{-ax}$ (n being a positive integer and $a > 0$) $=$ _____

(A) – 1 (B) 0 (C) 1 (D) None of these

g) If $Q = r \cot\theta$, then $\frac{\partial Q}{\partial r}$ is equal to

(A) $\cot\theta$ (B) $-\operatorname{cosec}^2\theta$ (C) $\cot\theta - r\operatorname{cosec}^2\theta$ (D) $\frac{1}{2}\cot\theta$

h) If $u = ax^2 + 2hxy + by^2$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{dy}$ is equal to

(A) 2u (B) u (C) 0 (D) none of these

i) If $f(x,y)=0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{\partial f/\partial x}{\partial f/\partial y}$ (B) $\frac{\partial f/\partial y}{\partial f/\partial x}$ (C) $-\frac{\partial f/\partial y}{\partial f/\partial x}$ (D) $-\frac{\partial f/\partial x}{\partial f/\partial y}$

j) If $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)}$ is equal to

(A) 1 (B) – 1 (C) 0 (D) none of these

k) If $y = \cos\theta + i\sin\theta$, then the value of $y + \frac{1}{y}$ is

(A) $2\cos\theta$ (B) $2\sin\theta$ (C) $2\operatorname{cosec}\theta$ (D) $2\tan\theta$

l) The number of solutions to the equation $z^2 + \bar{z} = 0$ is

(A) 1 (B) 2 (C) 3 (D) 4

m) If A is a non-zero column vector $(n \times 1)$, then the rank of matrix AA^T is

(A) 0 (B) 1 (C) $n-1$ (D) n

n) The matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ is given. The eigenvalues of $4A^{-1} + 3A + 2I$ are

(A) 6, 15 (B) 9, 12 (C) 9, 15 (D) 7, 15

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) If $y = \frac{x}{x^2 + a^2}$ then find y_n. (5)

b) Expand $f(x) = \frac{e^x}{e^x + 1}$ in powers of x up to x^3 by Maclaurin's series. (5)

c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that (4)

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$$

Q-3 Attempt all questions (14)

a) If $y = \sin(m\sin^{-1}x)$ then prove that (5)

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$

b) Prove that $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - .....$ (5)

c) Evaluate: $\lim_{x \to 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ (4)

Q-4 Attempt all questions (14)

a) Evaluate: $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$ **(5)**

b) If $u=\frac{y^2}{x}$, $v=\frac{x^2}{y}$, evaluate $J=\left(\frac{x,y}{u,v}\right)$ and $J'=\left(\frac{u,v}{x,y}\right)$ and hence verify that $JJ'=1$. **(5)**

c) Expand $f(x)=x^4-11x^3+43x^2-60x+14$ in powers of $(x-3)$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) If $u=\sec^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then find $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$. **(5)**

b) Evaluate: $\lim_{x\to 0}\frac{a}{x^2}\left[\frac{\sin kx}{\sin lx}-\frac{k}{l}\right]$ **(5)**

c) Find n[th] derivative of $\tan^{-1}x$. **(4)**

Q-6 **Attempt all questions** **(14)**

a) Using the formula $R=\frac{E}{I}$, find the maximum error and percentage of error in R if $I=20$ with a possible error of 0.1 and $E=120$ with a possible error of 0.05 and $R=6$. **(5)**

b) Find the continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. **(5)**

c) Verify Caley-Hamilton theorem for the matrix $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Find the inverse of $A=\begin{bmatrix}-1&1&2\\3&-1&1\\-1&3&4\end{bmatrix}$ by Gauss-Jordan reduction method. **(5)**

b) Find the fourth roots of unity and sketch them on the unit circle. **(5)**

c) If $\tan(\alpha+i\beta)=x+iy$ then prove that $x^2+y^2+2x\cot 2\alpha=1$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Investigate for what values of λ and μ the equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(5)**

b) If $x_r=\cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$ then prove that $\lim_{n\to\infty}x_1x_2x_3\ldots x_n=-1$. **(5)**

c) Check whether the following set of vectors is linearly dependent or linearly independent: **(4)**

$(1,0,1),(1,1,0),(1,-1,1),(1,2,-3)$